A most ingenious paradox!
We’ve quips and quibbles heard in flocks,
But none to beat this paradox!
A paradox, a paradox,
A most ingenious paradox!

—Gilbert and Sullivan, *The Pirates of Penzance*

**A MOST HAZARDOUS PARADOX**

**Milton Rosenstein**

**SING** words to think or communicate is a hazardous occupation. Words are volatile entities contaminated by numerous shifting implications. We do not just *use* words. Often, words use *us*. Words control and direct our thoughts and predetermine our conclusions. At least, when unaware, we allow words to do this.

But how can this happen? Perhaps some are aware of the danger and can guard against it. Most language users are not. Teachers, writers, speech makers, reporters, authors, philosophers, etc., whose effectiveness depends on their words, are often only dimly aware of the problem and go through life allowing themselves to be misused by words, spreading misunderstandings and making plans which don’t work, as well as drowning the world in pointless argumentation, error, and unsupportable opinion.

Have I exaggerated the hazards of mindless verbalizing? Just look at the volume of conflicting opinions about almost everything and the apparent impossibility of resolving most differences.

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The study of general semantics is concerned with, among other things, the effects of language upon thinking and communication. General semantics shows how language entraps the mind; the discipline also suggests strategies to avoid such traps.

Though it may seem strange, we start our discussion of the meanings of statements by looking at paradoxes. To look into the scope of meaningful statements we must understand why some statements are meaningless. The dictionary definition of a paradox is very broad:

1. a statement that seems contrary to common belief,

2. a statement that seems contradictory, unbelievable, or absurd but may actually be true in fact,

3. a statement that is self-contradictory in fact and hence false.

There is no shortage of similar definitions. In fact the whole field of definitions, explanations, and rationalizations of paradoxes is an impenetrable morass that is definitely beyond my patience. What we need is a broad, general understanding of the structure of the paradox.

Interest in exploring paradox is widespread. It is safe to say that paradox preoccupies most mathematicians because mathematical proofs are based on infinite sets that contain possible contradictions which are resolved by the use of involved and questionable solutions which often involve or skirt the borders of paradox.

For our purposes, we set aside the difficulties of mathematical proofs and approach our task by first noting that a meaningful statement or locution must have a structure that corresponds with reality. Korzybski held that “words are not things,” but to effectively represent reality words must “match” things. If a match does not exist, we cannot use such a statement, which in some cases may constitute a paradox. We can establish a rule for deciding whether or not to accept the statement.

A broad classification of paradoxes might include:

1. Paradoxes that fail to match reality because necessary elements are omitted.

2. Paradoxes that are based on assumptions about reality that are incorrect.


4. Paradoxes that play on words.

Some examples follow:
Zeno’s Tortoise

Zeno’s paradox, formulated about 425 BC, is a good example of a paradox that fails because a critical structure is omitted.

Zeno’s paradox holds that in a race between Achilles and a tortoise, if the tortoise is initially given a lead of any length, Achilles will never reach or pass the tortoise. This claim puzzled philosophers for many years. It was reasoned that Achilles must first cut the tortoise’ lead in half, but after he has done so, there still remains another separation which again must be cut in half before Achilles could reach the tortoise. This process goes on forever. Therefore, Achilles can never catch up with the tortoise.

This paradox exists because of the omission of a critical element of structure, time. Once time is incorporated there are numerous ways to show that Achilles would indeed pass the tortoise. If the tortoise is 20 feet ahead of Achilles, who is traveling 5 feet-per-minute faster than the tortoise, Achilles will reach the tortoise in 4 minutes and then pass it. Time has been excluded: we must remember that whenever we communicate, something gets left out.

Including time, we may “resolve” the paradox in another way: If Achilles must repeatedly cut the distance to the tortoise in half, the time it takes to do this is also cut in half. Ultimately, for practical purposes, this interval will equal zero and can be disregarded. Thus, Achilles reaches and then passes the tortoise.

The first explanation is the simplest. We have recast the paradox so that its structure is clear. The second explanation incorporates unneeded and complicating implications. How does the time interval actually become zero instead of just approaching it? How long does this actually take? How exactly does this correspond with physical reality?

Zeno’s Heap

Another paradox that is built upon a questionable view of the real world is Zeno’s question about constructing a heap. If, he asks, we start with a grain of sand which has essentially no mass and add more grains of the same size, how can we ultimately create a heap? This problem becomes more challenging if the particles become geometrical points which are lined up one after the other. How can they form a line? The structural error in both cases is an incorrect view of reality. In the first case a grain of sand cannot have no mass, and in the second we have accepted a mathematical assumption as physically real.

Many assumptions about structures are dreamt up by mathematicians in order to solve or clarify problems. If they accomplish this, the “solutions” are
gratefully accepted, but no one should claim, think, or even care if such solutions are part of physical reality. This procedure, however, is typical and is the reason that mathematics is crammed with paradoxes.

A source of mathematical paradoxes which has been transferred to the real world involves paradoxes of self-reference. For example:

- “All Cretans are liars,” says the Cretan.
- Russell’s barber, who only shaves those people who do not shave themselves.
- The librarian who puts together a list to be kept in her library of all bibliographies in her library that do not list themselves.

The hue and cry about of self-reference paradoxes reflects an attempt to make language do what it is unable to do well. Perhaps these arguments are residues of those heated discussions by Whitehead and Russell over the Theory of Types and concern universal propositions which have unrestricted ranges of values which the Theory attempts to resolve. We can escape this furor by encouraging theoreticians to restrict the scope of their meanings or to clarify the problems they wish to solve.

The contradictions in Russell’s paradox of the barber who only shaves those people who do not shave themselves can be settled by stipulating more about the barber. Some solutions exist.

1. She, the barber, doesn’t need to shave.
2. He, the barber, has an enormous beard.

A third example of a well-known paradox is the paradox that exists because of word confusion. Consider the brain teaser that asks:

When a tree falls in a forest in which there are no living creatures, does it make a sound?

The confusion arises because there is more than one meaning for the word “sound.” One meaning refers to the vibration of air molecules due to a perturbation of the air. Another refers to an event in the ears of a living creature that is induced by a perturbation in the air reaching it and is then interpreted by the nervous system. Incidentally, there are many such cases where one word has many meanings.

Paradoxes are abundant in the English language and for many people it is merely an amusing game to form them or to explain them away. However, there is a deeper lesson to be learned from the study of paradoxes. We have seen that
for any useful representation to describe reality, its structure must correspond with this reality. For a road map to be useful, its structure must match the structure of the territory it represents. Similarly, to be useful, a verbal description must have a structure that corresponds with the territory.

It is possible to form language depictions of things which do not exist and which cannot exist. A graphical representation can show an architectural structure which cannot exist, as in the work of M. C. Escher. Such a depiction is correctly termed paradoxical. Verbal paradoxes are often just this, verbal delineations of structures which cannot exist.

The abundance of verbal paradoxes in English reveals a dangerous pitfall in our use of language. Apparently it is quite possible to think, talk, or write about meaningless nothings, about situations that can never exist, although real-world physical paradoxes are clearly impossible. Paradoxes reveal the gulf between language and reality, between the “map,” and the “territory.”

In the “territory,” paradoxes cannot exist. If we were aware of this, we would not get so upset over them. Philosophers would not have worked hundreds, sometimes thousands of years, to resolve a paradox. Scientists would not become upset over such formulations as Russell’s paradoxes of self reference. They would realize that language is an “artificial creation,” not necessarily connected to reality, and as such it can be manipulated and twisted to do almost anything.

Paradoxes can be useful. We can “explain” a paradox by demonstrating that its verbal assertion has little chance of being logically true or of matching any real structure.

Rules of Operation

The evaluation of assertions is fraught with difficulty. Rather than employ a language such as English, we can more effectively start with a less ambiguous language such as mathematics. Unlike English, in mathematics the fundamental postulates, actually assumptions, and rules of operations, are explicit. These postulates are usually simple and every effort is made to make them simpler, clearly defined, and fewer. We usually state them without proof and we delineate the environment in which the given mathematics operates.

A basic postulate of arithmetic says 1+1= 2. This establishes the domain in which the postulate works and defines it as a linear math. Therefore arithmetic is only applicable in a world of linear relations, which includes adding, multiplying, and several other operations. The mixing of one gallon of water with another gallon of water, a linear operation, is correctly handled by arithmetic which predicts two gallons as a result, a true result. However mixing a gallon of
water with a gallon of gasoline is a nonlinear operation giving less than two gallons of mixture, and so arithmetic does not work here.

The statement 1+1=2 is logical for both cases given above, but is true for the first case, and false for the second. This demonstrates the limitation of logic, which only signifies that the expression does, or does not, violate the rules of a particular language, but has nothing to do with its truth or falsity. This can only be established by actual or previous experimentation with mixing water and gasoline.

The above may shock those who strongly believe there is an intimate connection between logic and truth. Since this is not so, nothing in either mathematical or verbal statements can be proven or disproven by logic. Unfortunately most people do not acknowledge this and continue to judge the truth of all assertions by the quality of their logic.